the wall is the curve lying on the surface $z = f(x, y)$. (We do not display the surface formed by the graph of $f$ in the figure, only the curve on it that is cut out by the cylinder.) From the definition

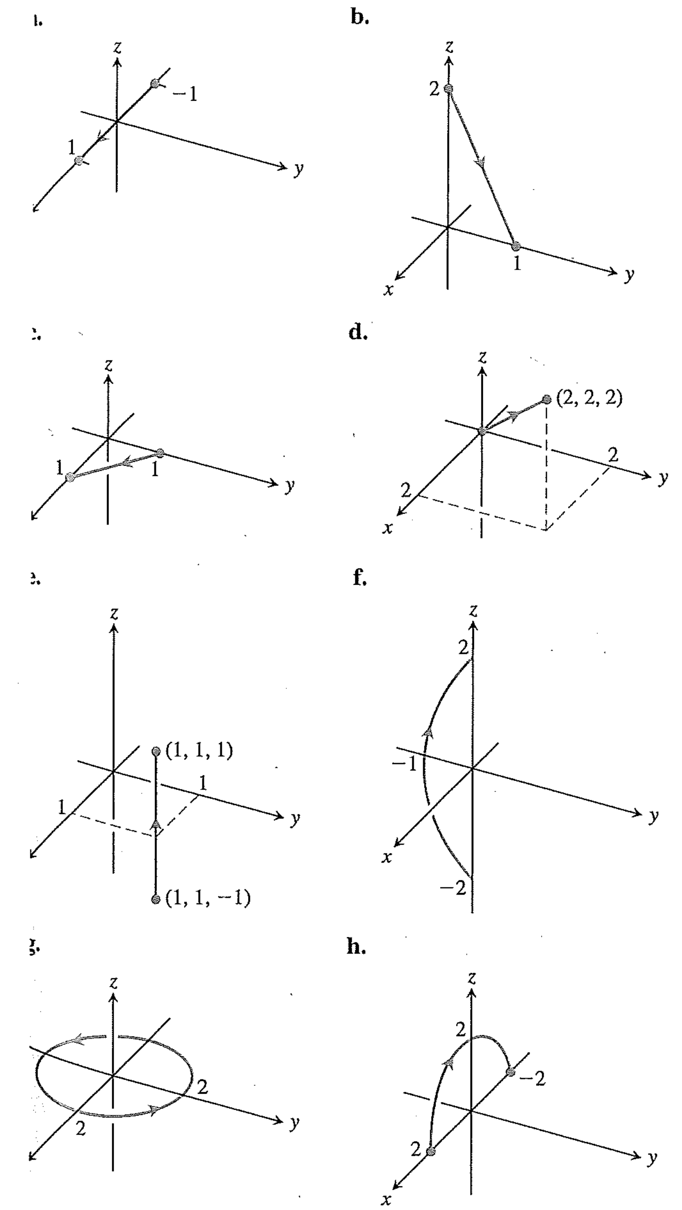
$$\int_C f\,ds = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k, y_k)\,\Delta s_k,$$

where $\Delta s_k \to 0$ as $n \to \infty$, we see that the line integral $\int_C f\,ds$ is the area of the wall shown in the figure.

## xercises 16.1

### aphs of Vector Equations

atch the vector equations in Exercises 1–8 with the graphs (a)–(h) ven here.



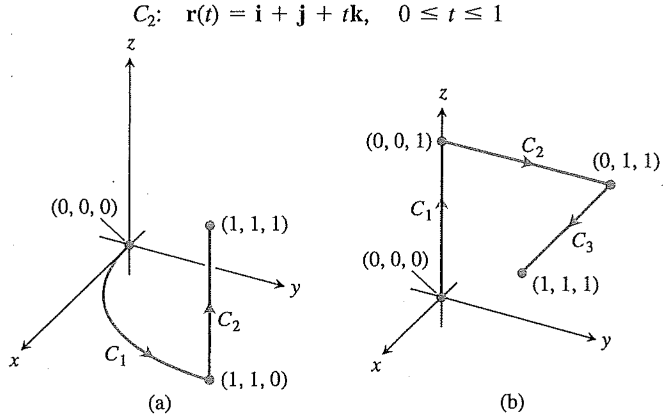
1. $\mathbf{r}(t) = t\mathbf{i} + (1 - t)\mathbf{j}, \quad 0 \le t \le 1$
2. $\mathbf{r}(t) = \mathbf{i} + \mathbf{j} + t\mathbf{k}, \quad -1 \le t \le 1$
3. $\mathbf{r}(t) = (2\cos t)\mathbf{i} + (2\sin t)\mathbf{j}, \quad 0 \le t \le 2\pi$
4. $\mathbf{r}(t) = t\mathbf{i}, \quad -1 \le t \le 1$
5. $\mathbf{r}(t) = t\mathbf{i} + t\mathbf{j} + t\mathbf{k}, \quad 0 \le t \le 2$
6. $\mathbf{r}(t) = t\mathbf{j} + (2 - 2t)\mathbf{k}, \quad 0 \le t \le 1$
7. $\mathbf{r}(t) = (t^2 - 1)\mathbf{j} + 2t\mathbf{k}, \quad -1 \le t \le 1$
8. $\mathbf{r}(t) = (2\cos t)\mathbf{i} + (2\sin t)\mathbf{k}, \quad 0 \le t \le \pi$

### Evaluating Line Integrals over Space Curves

9. Evaluate $\int_C (x + y)\,ds$ where $C$ is the straight-line segment $x = t, y = (1 - t), z = 0$, from $(0, 1, 0)$ to $(1, 0, 0)$.
10. Evaluate $\int_C (x - y + z - 2)\,ds$ where $C$ is the straight-line segment $x = t, y = (1 - t), z = 1$, from $(0, 1, 1)$ to $(1, 0, 1)$.
11. Evaluate $\int_C (xy + y + z)\,ds$ along the curve $\mathbf{r}(t) = 2t\mathbf{i} + t\mathbf{j} + (2 - 2t)\mathbf{k}, 0 \le t \le 1$.
12. Evaluate $\int_C \sqrt{x^2 + y^2}\,ds$ along the curve $\mathbf{r}(t) = (4\cos t)\mathbf{i} + (4\sin t)\mathbf{j} + 3t\mathbf{k}, -2\pi \le t \le 2\pi$.
13. Find the line integral of $f(x, y, z) = x + y + z$ over the straight-line segment from $(1, 2, 3)$ to $(0, -1, 1)$.
14. Find the line integral of $f(x, y, z) = \sqrt{3}/(x^2 + y^2 + z^2)$ over the curve $\mathbf{r}(t) = t\mathbf{i} + t\mathbf{j} + t\mathbf{k}, 1 \le t \le \infty$.
15. Integrate $f(x, y, z) = x + \sqrt{y} - z^2$ over the path from $(0, 0, 0)$ to $(1, 1, 1)$ (see accompanying figure) given by

$C_1$: $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j}, \quad 0 \le t \le 1$

$C_2$: $\mathbf{r}(t) = \mathbf{i} + \mathbf{j} + t\mathbf{k}, \quad 0 \le t \le 1$



The paths of integration for Exercises 15 and 16.

16. Integrate $f(x, y, z) = x + \sqrt{y} - z^2$ over the path from (0, 0, 0) to (1, 1, 1) (see accompanying figure) given by

$$C_1: \ \mathbf{r}(t) = t\mathbf{k}, \quad 0 \le t \le 1$$
$$C_2: \ \mathbf{r}(t) = t\mathbf{j} + \mathbf{k}, \quad 0 \le t \le 1$$
$$C_3: \ \mathbf{r}(t) = t\mathbf{i} + \mathbf{j} + \mathbf{k}, \quad 0 \le t \le 1$$

17. Integrate $f(x, y, z) = (x + y + z)/(x^2 + y^2 + z^2)$ over the path $\mathbf{r}(t) = t\mathbf{i} + t\mathbf{j} + t\mathbf{k}, 0 < a \le t \le b.$

18. Integrate $f(x, y, z) = -\sqrt{x^2 + z^2}$ over the circle

$$\mathbf{r}(t) = (a \cos t)\mathbf{j} + (a \sin t)\mathbf{k}, \qquad 0 \le t \le 2\pi.$$

### Line Integrals over Plane Curves

19. Evaluate $\int_C x\,ds$, where $C$ is
   a. the straight-line segment $x = t, y = t/2$, from (0, 0) to (4, 2).
   b. the parabolic curve $x = t, y = t^2$, from (0, 0) to (2, 4).

20. Evaluate $\int_C \sqrt{x + 2y}\,ds$, where $C$ is
   a. the straight-line segment $x = t, y = 4t$, from (0, 0) to (1, 4).
   b. $C_1 \cup C_2$; $C_1$ is the line segment from (0, 0) to (1, 0) and $C_2$ is the line segment from (1, 0) to (1, 2).

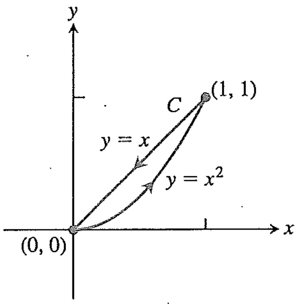
21. Find the line integral of $f(x, y) = ye^{x^2}$ along the curve $\mathbf{r}(t) = 4t\mathbf{i} - 3t\mathbf{j}, -1 \le t \le 2.$

22. Find the line integral of $f(x, y) = x - y + 3$ along the curve $\mathbf{r}(t) = (\cos t)\mathbf{i} + (\sin t)\mathbf{j}, 0 \le t \le 2\pi.$

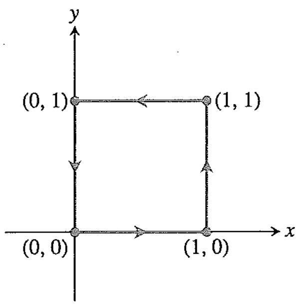
23. Evaluate $\displaystyle\int_C \frac{x^2}{y^{4/3}}\,ds$, where $C$ is the curve $x = t^2, y = t^3$, for $1 \le t \le 2.$

24. Find the line integral of $f(x, y) = \sqrt{y}/x$ along the curve $\mathbf{r}(t) = t^3\mathbf{i} + t^4\mathbf{j}, 1/2 \le t \le 1.$

25. Evaluate $\int_C \left(x + \sqrt{y}\right) ds$ where $C$ is given in the accompanying figure.




26. Evaluate $\displaystyle\int_C \frac{1}{x^2 + y^2 + 1}\,ds$ where $C$ is given in the accompanying figure.




In Exercises 27–30, integrate $f$ over the given curve.

27. $f(x, y) = x^3/y, \quad C: \ y = x^2/2, \quad 0 \le x \le 2$

28. $f(x, y) = (x + y^2)/\sqrt{1 + x^2}, \quad C: \ y = x^2/2$ from (1, 1/2) to (0, 0)

29. $f(x, y) = x + y, \quad C: \ x^2 + y^2 = 4$ in the first quadrant from (2, 0) to (0, 2)

30. $f(x, y) = x^2 - y, \quad C: \ x^2 + y^2 = 4$ in the first quadrant from (0, 2) to $\left(\sqrt{2}, \sqrt{2}\right)$

31. Find the area of one side of the "winding wall" standing orthogonally on the curve $y = x^2, 0 \le x \le 2$, and beneath the curve on the surface $f(x, y) = x + \sqrt{y}$.

32. Find the area of one side of the "wall" standing orthogonally on the curve $2x + 3y = 6, 0 \le x \le 6$, and beneath the curve on the surface $f(x, y) = 4 + 3x + 2y$.

### Masses and Moments

33. **Mass of a wire** Find the mass of a wire that lies along the curve $\mathbf{r}(t) = (t^2 - 1)\mathbf{j} + 2t\mathbf{k}, 0 \le t \le 1$, if the density is $\delta = (3/2)t$.

34. **Center of mass of a curved wire** A wire of density $\delta(x, y, z) = 15\sqrt{y + 2}$ lies along the curve $\mathbf{r}(t) = (t^2 - 1)\mathbf{j} + 2t\mathbf{k}, -1 \le t \le 1$. Find its center of mass. Then sketch the curve and center of mass together.

35. **Mass of wire with variable density** Find the mass of a thin wire lying along the curve $\mathbf{r}(t) = \sqrt{2}t\mathbf{i} + \sqrt{2}t\mathbf{j} + (4 - t^2)\mathbf{k}$, $0 \le t \le 1$, if the density is (a) $\delta = 3t$ and (b) $\delta = 1$.

36. **Center of mass of wire with variable density** Find the center of mass of a thin wire lying along the curve $\mathbf{r}(t) = t\mathbf{i} + 2t\mathbf{j} + (2/3)t^{3/2}\mathbf{k}, 0 \le t \le 2$, if the density is $\delta = 3\sqrt{5 + t}$.

37. **Moment of inertia of wire hoop** A circular wire hoop of constant density $\delta$ lies along the circle $x^2 + y^2 = a^2$ in the $xy$-plane. Find the hoop's moment of inertia about the $z$-axis.

38. **Inertia of a slender rod** A slender rod of constant density lies along the line segment $\mathbf{r}(t) = t\mathbf{j} + (2 - 2t)\mathbf{k}, 0 \le t \le 1$, in the $yz$-plane. Find the moments of inertia of the rod about the three coordinate axes.

39. **Two springs of constant density** A spring of constant density $\delta$ lies along the helix

$$\mathbf{r}(t) = (\cos t)\mathbf{i} + (\sin t)\mathbf{j} + t\mathbf{k}, \qquad 0 \le t \le 2\pi.$$

   a. Find $I_z$.
   b. Suppose that you have another spring of constant density $\delta$ that is twice as long as the spring in part (a) and lies along the helix for $0 \le t \le 4\pi$. Do you expect $I_z$ for the longer spring to be the same as that for the shorter one, or should it be different? Check your prediction by calculating $I_z$ for the longer spring.

40. **Wire of constant density** A wire of constant density $\delta = 1$ lies along the curve

$$\mathbf{r}(t) = (t \cos t)\mathbf{i} + (t \sin t)\mathbf{j} + \left(2\sqrt{2}/3\right)t^{3/2}\mathbf{k}, \qquad 0 \le t \le 1.$$

Find $\bar{z}$ and $I_z$.

41. **The arch in Example 4** Find $I_x$ for the arch in Example 4.

## Exercises 16.2

### Vector Fields

Find the gradient fields of the functions in Exercises 1–4.

1. $f(x, y, z) = (x^2 + y^2 + z^2)^{-1/2}$
2. $f(x, y, z) = \ln\sqrt{x^2 + y^2 + z^2}$
3. $g(x, y, z) = e^z - \ln(x^2 + y^2)$
4. $g(x, y, z) = xy + yz + xz$
5. Give a formula $\mathbf{F} = M(x, y)\mathbf{i} + N(x, y)\mathbf{j}$ for the vector field in the plane that has the property that $\mathbf{F}$ points toward the origin with magnitude inversely proportional to the square of the distance from $(x, y)$ to the origin. (The field is not defined at $(0, 0)$.)
6. Give a formula $\mathbf{F} = M(x, y)\mathbf{i} + N(x, y)\mathbf{j}$ for the vector field in the plane that has the properties that $\mathbf{F} = \mathbf{0}$ at $(0, 0)$ and that at any other point $(a, b)$, $\mathbf{F}$ is tangent to the circle $x^2 + y^2 = a^2 + b^2$ and points in the clockwise direction with magnitude $|\mathbf{F}| = \sqrt{a^2 + b^2}$.

### Line Integrals of Vector Fields

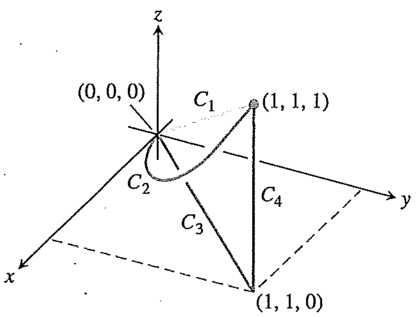
In Exercises 7–12, find the line integrals of $\mathbf{F}$ from $(0, 0, 0)$ to $(1, 1, 1)$ over each of the following paths in the accompanying figure.

a. The straight-line path $C_1$: $\mathbf{r}(t) = t\mathbf{i} + t\mathbf{j} + t\mathbf{k}$, $0 \le t \le 1$

b. The curved path $C_2$: $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + t^4\mathbf{k}$, $0 \le t \le 1$

c. The path $C_3 \cup C_4$ consisting of the line segment from $(0, 0, 0)$ to $(1, 1, 0)$ followed by the segment from $(1, 1, 0)$ to $(1, 1, 1)$

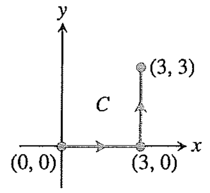
7. $\mathbf{F} = 3y\mathbf{i} + 2x\mathbf{j} + 4z\mathbf{k}$
8. $\mathbf{F} = [1/(x^2 + 1)]\mathbf{j}$
9. $\mathbf{F} = \sqrt{z}\mathbf{i} - 2x\mathbf{j} + \sqrt{y}\mathbf{k}$
10. $\mathbf{F} = xy\mathbf{i} + yz\mathbf{j} + xz\mathbf{k}$
11. $\mathbf{F} = (3x^2 - 3x)\mathbf{i} + 3z\mathbf{j} + \mathbf{k}$
12. $\mathbf{F} = (y + z)\mathbf{i} + (z + x)\mathbf{j} + (x + y)\mathbf{k}$



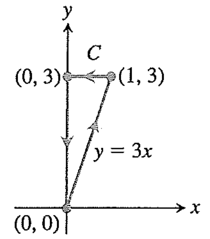

### Line Integrals with Respect to $x$, $y$, and $z$

In Exercises 13–16, find the line integrals along the given path $C$.

13. $\int_C (x - y)\,dx$, where $C$: $x = t$, $y = 2t + 1$, for $0 \le t \le 3$
14. $\int_C \frac{x}{y}\,dy$, where $C$: $x = t$, $y = t^2$, for $1 \le t \le 2$
15. $\int_C (x^2 + y^2)\,dy$, where $C$ is given in the accompanying figure




16. $\int_C \sqrt{x + y}\,dx$, where $C$ is given in the accompanying figure




17. Along the curve $\mathbf{r}(t) = t\mathbf{i} - \mathbf{j} + t^2\mathbf{k}$, $0 \le t \le 1$, evaluate each of the following integrals.

 a. $\int_C (x + y - z)\,dx$  b. $\int_C (x + y - z)\,dy$

 c. $\int_C (x + y - z)\,dz$

18. Along the curve $\mathbf{r}(t) = (\cos t)\mathbf{i} + (\sin t)\mathbf{j} - (\cos t)\mathbf{k}$, $0 \le t \le \pi$, evaluate each of the following integrals.

 a. $\int_C xz\,dx$  b. $\int_C xz\,dy$  c. $\int_C xyz\,dz$

### Work

In Exercises 19–22, find the work done by $\mathbf{F}$ over the curve in the direction of increasing $t$.

19. $\mathbf{F} = xy\mathbf{i} + y\mathbf{j} - yz\mathbf{k}$
 $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + t\mathbf{k}$, $0 \le t \le 1$
20. $\mathbf{F} = 2y\mathbf{i} + 3x\mathbf{j} + (x + y)\mathbf{k}$
 $\mathbf{r}(t) = (\cos t)\mathbf{i} + (\sin t)\mathbf{j} + (t/6)\mathbf{k}$, $0 \le t \le 2\pi$
21. $\mathbf{F} = z\mathbf{i} + x\mathbf{j} + y\mathbf{k}$
 $\mathbf{r}(t) = (\sin t)\mathbf{i} + (\cos t)\mathbf{j} + t\mathbf{k}$, $0 \le t \le 2\pi$
22. $\mathbf{F} = 6z\mathbf{i} + y^2\mathbf{j} + 12x\mathbf{k}$
 $\mathbf{r}(t) = (\sin t)\mathbf{i} + (\cos t)\mathbf{j} + (t/6)\mathbf{k}$, $0 \le t \le 2\pi$

### Line Integrals in the Plane

23. Evaluate $\int_C xy\,dx + (x + y)\,dy$ along the curve $y = x^2$ from $(-1, 1)$ to $(2, 4)$.
24. Evaluate $\int_C (x - y)\,dx + (x + y)\,dy$ counterclockwise around the triangle with vertices $(0, 0)$, $(1, 0)$, and $(0, 1)$.
25. Evaluate $\int_C \mathbf{F}\cdot\mathbf{T}\,ds$ for the vector field $\mathbf{F} = x^2\mathbf{i} - y\mathbf{j}$ along the curve $x = y^2$ from $(4, 2)$ to $(1, -1)$.
26. Evaluate $\int_C \mathbf{F}\cdot d\mathbf{r}$ for the vector field $\mathbf{F} = y\mathbf{i} - x\mathbf{j}$ counterclockwise along the unit circle $x^2 + y^2 = 1$ from $(1, 0)$ to $(0, 1)$.

ılation, and Flux in the Plane

Find the work done by the force $\mathbf{F} = xy\mathbf{i} + (y - x)\mathbf{j}$ ɜ straight line from (1, 1) to (2, 3).

Find the work done by the gradient of $f(x, y) = (x + y)^2$ ·clockwise around the circle $x^2 + y^2 = 4$ from (2, 0) to

**ation and flux** Find the circulation and flux of the fields

$$\mathbf{F}_1 = x\mathbf{i} + y\mathbf{j} \quad \text{and} \quad \mathbf{F}_2 = -y\mathbf{i} + x\mathbf{j}$$

and across each of the following curves.

: circle $\mathbf{r}(t) = (\cos t)\mathbf{i} + (\sin t)\mathbf{j}, \quad 0 \le t \le 2\pi$

: ellipse $\mathbf{r}(t) = (\cos t)\mathbf{i} + (4 \sin t)\mathbf{j}, \quad 0 \le t \le 2\pi$

**cross a circle** Find the flux of the fields

$$_1 = 2x\mathbf{i} - 3y\mathbf{j} \quad \text{and} \quad \mathbf{F}_2 = 2x\mathbf{i} + (x - y)\mathbf{j}$$

the circle

$$\mathbf{r}(t) = (a \cos t)\mathbf{i} + (a \sin t)\mathbf{j}, \qquad 0 \le t \le 2\pi.$$

ːs 31–34, find the circulation and flux of the field **F** around the closed semicircular path that consists of the semicircu- $(t) = (a \cos t)\mathbf{i} + (a \sin t)\mathbf{j}$, $0 \le t \le \pi$, followed by the nt $\mathbf{r}_2(t) = t\mathbf{i}, -a \le t \le a$.

i + yj **32.** $\mathbf{F} = x^2\mathbf{i} + y^2\mathbf{j}$

-yi + xj **34.** $\mathbf{F} = -y^2\mathbf{i} + x^2\mathbf{j}$

**integrals** Find the flow of the velocity field $\mathbf{F} =$ ·)i $- (x^2 + y^2)\mathbf{j}$ along each of the following paths from to (−1, 0) in the *xy*-plane.

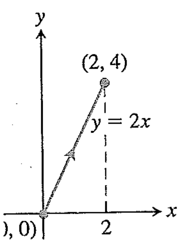
e upper half of the circle $x^2 + y^2 = 1$

e line segment from (1, 0) to (−1, 0)

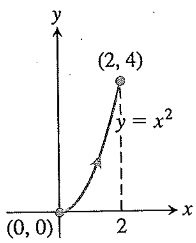
e line segment from (1, 0) to (0, −1) followed by the line ɡment from (0, −1) to (−1, 0)

**across a triangle** Find the flux of the field **F** in Exercise :ward across the triangle with vertices (1, 0), (0, 1), (−1, 0).

he flow of the velocity field $\mathbf{F} = y^2\mathbf{i} + 2xy\mathbf{j}$ along each of llowing paths from (0, 0) to (2, 4).

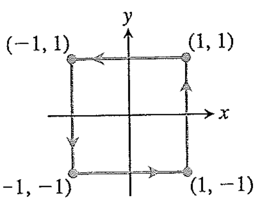



**b.**

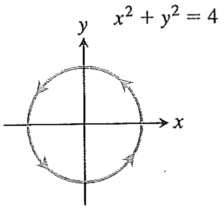



se any path from (0, 0) to (2, 4) different from parts (a) ıd (b).

the circulation of the field $\mathbf{F} = y\mathbf{i} + (x + 2y)\mathbf{j}$ around each : following closed paths.




**b.**




**c.** Use any closed path different from parts (a) and (b).

Vector Fields in the Plane

**39. Spin field** Draw the spin field

$$\mathbf{F} = -\frac{y}{\sqrt{x^2 + y^2}}\mathbf{i} + \frac{x}{\sqrt{x^2 + y^2}}\mathbf{j}$$

(see Figure 16.12) along with its horizontal and vertical components at a representative assortment of points on the circle $x^2 + y^2 = 4$.

**40. Radial field** Draw the radial field

$$\mathbf{F} = x\mathbf{i} + y\mathbf{j}$$

(see Figure 16.11) along with its horizontal and vertical components at a representative assortment of points on the circle $x^2 + y^2 = 1$.

**41. A field of tangent vectors**

**a.** Find a field $\mathbf{G} = P(x, y)\mathbf{i} + Q(x, y)\mathbf{j}$ in the *xy*-plane with the property that at any point $(a, b) \ne (0, 0)$, **G** is a vector of magnitude $\sqrt{a^2 + b^2}$ tangent to the circle $x^2 + y^2 = a^2 + b^2$ and pointing in the counterclockwise direction. (The field is undefined at (0, 0).)

**b.** How is **G** related to the spin field **F** in Figure 16.12?

**42. A field of tangent vectors**

**a.** Find a field $\mathbf{G} = P(x, y)\mathbf{i} + Q(x, y)\mathbf{j}$ in the *xy*-plane with the property that at any point $(a, b) \ne (0, 0)$, **G** is a unit vector tangent to the circle $x^2 + y^2 = a^2 + b^2$ and pointing in the clockwise direction.

**b.** How is **G** related to the spin field **F** in Figure 16.12?

**43. Unit vectors pointing toward the origin** Find a field $\mathbf{F} = M(x, y)\mathbf{i} + N(x, y)\mathbf{j}$ in the *xy*-plane with the property that at each point $(x, y) \ne (0, 0)$, **F** is a unit vector pointing toward the origin. (The field is undefined at (0, 0).)

**44. Two "central" fields** Find a field $\mathbf{F} = M(x, y)\mathbf{i} + N(x, y)\mathbf{j}$ in the *xy*-plane with the property that at each point $(x, y) \ne (0, 0)$, **F** points toward the origin and $|\mathbf{F}|$ is **(a)** the distance from $(x, y)$ to the origin, **(b)** inversely proportional to the distance from $(x, y)$ to the origin. (The field is undefined at (0, 0).)

**45. Work and area** Suppose that $f(t)$ is differentiable and positive for $a \le t \le b$. Let $C$ be the path $\mathbf{r}(t) = t\mathbf{i} + f(t)\mathbf{j}$, $a \le t \le b$, and $\mathbf{F} = y\mathbf{i}$. Is there any relation between the value of the work integral

$$\int_C \mathbf{F} \cdot d\mathbf{r}$$

and the area of the region bounded by the *t*-axis, the graph of *f*, and the lines $t = a$ and $t = b$? Give reasons for your answer.

**46. Work done by a radial force with constant magnitude** A particle moves along the smooth curve $y = f(x)$ from $(a, f(a))$ to

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**47.** F =
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$(b, f(b))$. The force moving the particle has constant magnitude $k$ and always points away from the origin. Show that the work done by the force is

$$\int_C \mathbf{F}\cdot\mathbf{T}\,ds = k\left[(b^2 + (f(b))^2)^{1/2} - (a^2 + (f(a))^2)^{1/2}\right].$$

### Flow Integrals in Space

In Exercises 47–50, $\mathbf{F}$ is the velocity field of a fluid flowing through a region in space. Find the flow along the given curve in the direction of increasing $t$.

**47.** $\mathbf{F} = -4xy\mathbf{i} + 8y\mathbf{j} + 2\mathbf{k}$
$\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + \mathbf{k}, \quad 0 \le t \le 2$

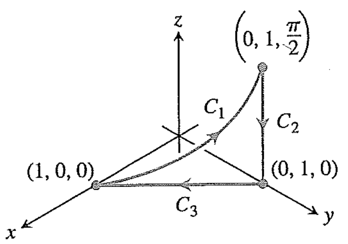
**48.** $\mathbf{F} = x^2\mathbf{i} + yz\mathbf{j} + y^2\mathbf{k}$
$\mathbf{r}(t) = 3t\mathbf{j} + 4t\mathbf{k}, \quad 0 \le t \le 1$

**49.** $\mathbf{F} = (x - z)\mathbf{i} + x\mathbf{k}$
$\mathbf{r}(t) = (\cos t)\mathbf{i} + (\sin t)\mathbf{k}, \quad 0 \le t \le \pi$

**50.** $\mathbf{F} = -y\mathbf{i} + x\mathbf{j} + 2\mathbf{k}$
$\mathbf{r}(t) = (-2\cos t)\mathbf{i} + (2\sin t)\mathbf{j} + 2t\mathbf{k}, \quad 0 \le t \le 2\pi$

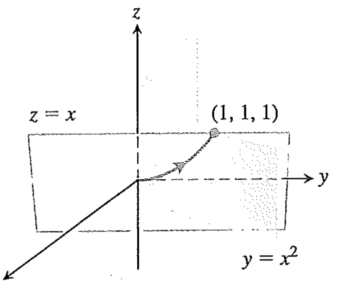
**51. Circulation** Find the circulation of $\mathbf{F} = 2x\mathbf{i} + 2z\mathbf{j} + 2y\mathbf{k}$ around the closed path consisting of the following three curves traversed in the direction of increasing $t$.

$C_1$: $\mathbf{r}(t) = (\cos t)\mathbf{i} + (\sin t)\mathbf{j} + t\mathbf{k}, \quad 0 \le t \le \pi/2$
$C_2$: $\mathbf{r}(t) = \mathbf{j} + (\pi/2)(1 - t)\mathbf{k}, \quad 0 \le t \le 1$
$C_3$: $\mathbf{r}(t) = t\mathbf{i} + (1 - t)\mathbf{j}, \quad 0 \le t \le 1$




**52. Zero circulation** Let $C$ be the ellipse in which the plane $2x + 3y - z = 0$ meets the cylinder $x^2 + y^2 = 12$. Show, without evaluating either line integral directly, that the circulation of the field $\mathbf{F} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ around $C$ in either direction is zero.

**53. Flow along a curve** The field $\mathbf{F} = xy\mathbf{i} + y\mathbf{j} - yz\mathbf{k}$ is the velocity field of a flow in space. Find the flow from $(0, 0, 0)$ to $(1, 1, 1)$ along the curve of intersection of the cylinder $y = x^2$ and the plane $z = x$. (*Hint:* Use $t = x$ as the parameter.)




**54. Flow of a gradient field** Find the flow of the field $\mathbf{F} = \nabla(xy^2z^3)$:

a. Once around the curve $C$ in Exercise 52, clockwise as viewed from above

b. Along the line segment from $(1, 1, 1)$ to $(2, 1, -1)$.

### COMPUTER EXPLORATIONS

In Exercises 55–60, use a CAS to perform the following steps for finding the work done by force $\mathbf{F}$ over the given path:

a. Find $d\mathbf{r}$ for the path $\mathbf{r}(t) = g(t)\mathbf{i} + h(t)\mathbf{j} + k(t)\mathbf{k}$.

b. Evaluate the force $\mathbf{F}$ along the path.

c. Evaluate $\int_C \mathbf{F}\cdot d\mathbf{r}$.

**55.** $\mathbf{F} = xy^6\mathbf{i} + 3x(xy^5 + 2)\mathbf{j}$; $\mathbf{r}(t) = (2\cos t)\mathbf{i} + (\sin t)\mathbf{j}$, $0 \le t \le 2\pi$

**56.** $\mathbf{F} = \dfrac{3}{1 + x^2}\mathbf{i} + \dfrac{2}{1 + y^2}\mathbf{j}$; $\mathbf{r}(t) = (\cos t)\mathbf{i} + (\sin t)\mathbf{j}$, $0 \le t \le \pi$

**57.** $\mathbf{F} = (y + yz\cos xyz)\mathbf{i} + (x^2 + xz\cos xyz)\mathbf{j} + (z + xy\cos xyz)\mathbf{k}$; $\mathbf{r}(t) = (2\cos t)\mathbf{i} + (3\sin t)\mathbf{j} + \mathbf{k}$, $0 \le t \le 2\pi$

**58.** $\mathbf{F} = 2xy\mathbf{i} - y^2\mathbf{j} + ze^x\mathbf{k}$; $\mathbf{r}(t) = -t\mathbf{i} + \sqrt{t}\mathbf{j} + 3t\mathbf{k}$, $1 \le t \le 4$

**59.** $\mathbf{F} = (2y + \sin x)\mathbf{i} + (z^2 + (1/3)\cos y)\mathbf{j} + x^4\mathbf{k}$; $\mathbf{r}(t) = (\sin t)\mathbf{i} + (\cos t)\mathbf{j} + (\sin 2t)\mathbf{k}, \quad -\pi/2 \le t \le \pi/2$

**60.** $\mathbf{F} = (x^2y)\mathbf{i} + \frac{1}{3}x^3\mathbf{j} + xy\mathbf{k}$; $\mathbf{r}(t) = (\cos t)\mathbf{i} + (\sin t)\mathbf{j} + (2\sin^2 t - 1)\mathbf{k}, \quad 0 \le t \le 2\pi$

# 16.3 Path Independence, Conservative Fields, and Potential Functions

A **gravitational field G** is a vector field that represents the effect of gravity at a point in space due to the presence of a massive object. The gravitational force on a body of mass $m$ placed in the field is given by $\mathbf{F} = m\mathbf{G}$. Similarly, an **electric field E** is a vector field in space that represents the effect of electric forces on a charged particle placed within it. The force on a body of charge $q$ placed in the field is given by $\mathbf{F} = q\mathbf{E}$. In gravitational and electric fields, the amount of work it takes to move a mass or charge from one point to another depends on the initial and final positions of the object—not on which path is taken between these positions. In this section we study vector fields with this property and the calculation of work integrals associated with them.